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**A REVIEW OF CRITICAL HEAT FLUX ENHANCEMENT IN NUCLEATE POOL  
BOILING OF NANOFLUID**

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**ABSTRACT**

Nucleate boiling is efficient heat transfer mode using the vaporization of a liquid. However, it is well known that there exists a critical value of the heat flux at which the heat transfer mechanism changes from the highly efficient nucleate boiling to extremely inefficient film boiling. This limiting heat flux is called Critical Heat Flux (CHF). CHF is the condition where the vapor generated by nucleate boiling becomes so large that it prevents the liquid from reaching and rewetting the surface, therefore it is an undesirable phenomenon causing an excessive increase of the temperature in the boiling phenomenon. It is important to enhance the CHF in order to improve the safety margin and economic performance in a thermal system. With the surge of nanofluids as potential candidates for cooling fluids, many studies have been reported on CHF enhancement using nanofluids. Nanofluids are colloidal dispersions of nanoparticle in a base fluid which is water, oil, bio-fluid and ethylene. Such nanofluids have the capability to enhance the CHF significantly about three times.

Over the past decade lots of experiments were performed on the nucleate boiling CHF of nanofluids. The objective of this paper is to provide an exhaustive review of these experiments. The effects of six parameters (i.e. Nanoparticle material, size, concentration, stabilizer effect, heater size and geometry, heater surface orientation) on CHF enhancement in nanofluids are systemically presented. Nanoparticle material which has shorter Rayleigh–Taylor wavelengths when coated on heater surface shows higher CHF enhancement. For nanoparticles size within nano range there is no significant variation in CHF. But comparing particle size within nano and micro range there is significant enhancement in CHF for particle size in nano range than particle size in micro range. With increasing nanoparticle concentrations CHF enhancement occurs up to certain concentration then CHF enhancement decreases with increase in nanoparticle concentration. There is lack of work which can give the effect of stabilizer on CHF. More work is required in this area. Higher value of CHF found on smaller heater size, also CHF on cylindrical wires is higher than that of the ribbons heaters. CHF values are higher for horizontal heater. CHF decreases as angle of inclination of heater surface increases. Finally, future research needs are identified.

**KEYWORDS:** Critical heat flux (CHF), Nanoparticle, Nanofluid, Concentration, stabilizer.

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**INTRODUCTION**

Nucleate boiling is a very efficient mode of heat transfer owing to the large energy required to realize the phase change from liquid to vapor. Therefore, several important industrial applications utilize nucleate boiling to remove large heat fluxes from hot surfaces. These include nuclear reactors, miniature electronic devices, refrigeration and cryogenic systems, chemical and thermal reactors, among others. However, it is well known that there exists a critical value of the heat flux at which the heat transfer mechanism changes from the highly efficient nucleate boiling to extremely inefficient film boiling

[2]. This limiting heat flux is called Critical Heat Flux (CHF). This is the highest heat flux where boiling heat transfer sustains its high cooling performance. When the surface reaches CHF, it becomes coated with a vapor film which isolates the heating surface and the fluid, and the heat transfer decreases dramatically. In these conditions, the wall temperature rises quickly, and if it exceeds the limits of its constituent materials, system failure occurs. Therefore, in most applications of boiling, the system is required to operate at power levels below that corresponding to CHF. For this reason, every system

incorporates a safety margin by running at a heat flux lower than CHF, but this approach reduces system efficiency. This compromise between safety and efficiency is a very serious problem in the industry. For this reason, a vast amount of work has been carried out to understand nucleate boiling CHF conditions, and to increase the CHF point [1]. A higher value of CHF allows for higher power density in thermal systems, which in turn makes these systems more compact and ultimately more economic. One way to increase CHF is to suspend a small amount of nanoparticles in the base fluid to form a suspension called nanofluid [2].

### CHF ENHANCEMENT

Several techniques to enhance the CHF have been explored. According to Rohsenow et al. [2] they can be classified into active (requiring external changes to the heater) or passive (requiring no external changes to the heater) methods. Typical active approaches include vibration of the heated surface or the cooling fluid (to increase the bubble departure frequency), heater rotation (to promote bubble departure from and liquid deposition onto the heater surface) and applying an external electric field (to facilitate the bubble departure from the surface), and passive approaches include coating the surface with porous coatings (to increase the number of active nucleation sites) and oxidation or selective fouling of heater surface (to increase surface hydrophilicity). A recent passive approach that has garnered increased attention worldwide is to create a colloidal suspension of solid nanoparticles in water or other base fluid, called nanofluids. There are various materials of choice for the dispersed nanoparticles—chemically stable metals (such as Cu, Au and Ag), metal oxides (such as Al<sub>2</sub>O<sub>3</sub>, SiO<sub>2</sub> and ZrO<sub>2</sub>) and different forms of carbon (such as diamond, graphite and fullerene). Boiling tests have shown these engineered nanofluid coolants to have a significantly higher CHF (generally, an enhancement ranging from 20% to more than 300%) compared to water [2].

Over the past decade, a considerable amount of research has been carried out in the area of nucleate boiling critical heat flux (CHF) in nanofluids. In the present paper review of studies published over the past decades is summarized in terms of the effects of six parameters on CHF enhancement in nanofluid boiling as follows:

1. Effect of Nanoparticle material on CHF
2. Effect of Nanoparticle size on CHF

3. Effect of Concentrations on CHF
4. Effect of Stabilizer on CHF
5. Effect of heater size and geometry on CHF
6. Effect of heater surface orientation on CHF

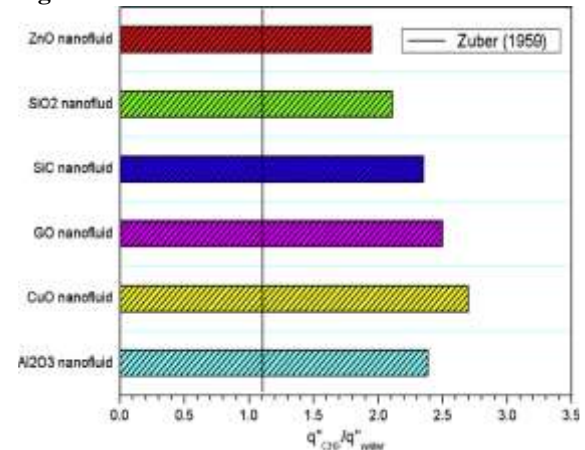
### Effect of Nanoparticle material on CHF

S.D. Park and I.C. Bang [3] used six types of Nanofluids with ZnO, SiO<sub>2</sub>, SiC, Al<sub>2</sub>O<sub>3</sub>, graphene oxide (GO) and CuO at 0.01% volume concentration in distilled water in order to find the effect of nanoparticle material on CHF.

At 0.01% volume concentration there was not significant change in the properties of dilute nanofluids compared with a base fluid, but the CHF in nanofluids was enhanced in comparison with distilled water. Figure 1 presents the CHF enhancement ratio for nanofluids. Each of the nanofluids has a different value. The CuO nanofluid shows the largest CHF enhancement of about 160%. The lowest CHF enhancement took place in the ZnO nanofluid, measured at 90%. Reasons for CHF enhancement in nanofluid compared with water on the basis of hydrodynamic instability theory [3].

The observed distance between the bubbles was different for each nanoparticle-coated surface. All of the nanoparticle-coated surfaces were having a shorter average distance between bubbles than the bare surface's bubbles. Fluids with high CHF enhancement exhibit short Rayleigh–Taylor wavelengths.

Figure 1:



Results of CHF enhancement for each test fluid.

Rayleigh–Taylor wavelengths observed for bare wire was 6.4mm, while for other nanoparticle coated heater was: ZnO coated wire 5.8mm, SiO<sub>2</sub> coated

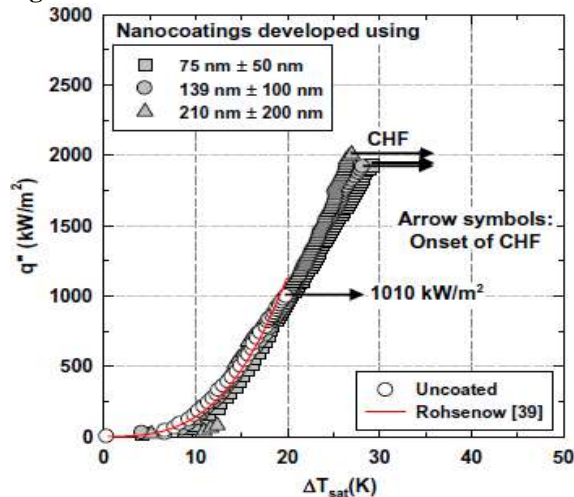
wire 5.6mm, SiC coated wire 5.2mm, Al<sub>2</sub>O<sub>3</sub> coated wire 5.2mm, GO coated wire 4.8mm and CuO coated wire 4.5mm. Therefore they conclude that when the Rayleigh–Taylor wavelength decreases, the CHF increases

Shortest and longest Rayleigh–Taylor wavelengths were observed for CuO and ZnO nanoparticles coated surface while for bare wire higher Rayleigh–Taylor wavelengths observed [3]. Hence highest CHF enhancement occurs in CuO nanofluid, while for ZnO nanofluid lowest CHF enhancement occurs. A short wavelength prevents the formation of a bulk of vapor by venting the vapor evenly across the heating element surface. Shorter wavelengths also increase wettability by allowing the liquid to break through the developing vapor film, which also enhances CHF. Due to shorter Rayleigh–Taylor wavelengths the number of bubble departure sites increases and the radius of the bubble reduce. Hence delay in liquid choking phenomena results in significant CHF enhanced.

### Effect of Nanoparticle size on CHF

S.M. Kwark et al. [14] conducted experiments to observe the role of average nanoparticle size on CHF. Uncoated and nanocoated test heater tested in pool boiling of nanofluids. Nanocoated surfaces were developed using Al<sub>2</sub>O<sub>3</sub> nanoparticle of three different sizes of 75nm, 139nm and 210 nm.

Figure 2:



Effect of nanoparticle size on CHF

The saturated pool boiling curves of the uncoated and the nanocoated surfaces at atmospheric pressure is shown in Figure 2. It is seen from curve that pool boiling curve of the nanocoated surfaces follows the same trend as that for the uncoated surface, but extends beyond and a dramatic CHF enhancement was observed. Under saturated pool boiling conditions, the magnitude of CHF enhancement is 80% and there is very negligible variation in CHF on nanocoated surfaces which were coated by different nanoparticle sizes. The SEM images of the nanocoatings showed no distinctive differences in the coating structures. Thus, over the range tested (75 nm, 139 nm and 210 nm), there is no significant dependence of CHF on the average size of the particles [14].

Silica Nanoparticles of sizes 15nm, 50nm and micro sized particles of 3 μm were used by Vassallo et al. [5] these particles were dispersed in deionized water at volume concentrations 0.5%. For both 15 and 50 nm particles boiling curves follow the pure water boiling curve throughout the nucleate boiling regime up to the CHF limit, then continue significantly higher CHF limit (i.e.60% higher) than deionized water. They found that The 50 nm silica solution allows a maximum heat flux about 3 times that of deionized water and nearly twice that allowed with the 3 μm silica solution.

From these results it is clear that particle size affect the CHF. Particle size within nano range there is no significant effect on the CHF as observed in case of 15 and 50 nm particle size, while comparing particle size in nano range and micro range there is vast difference in CHF. Nanoparticle shows significant CHF enhancement than particle size in micro range.

### Effect of Concentrations on CHF

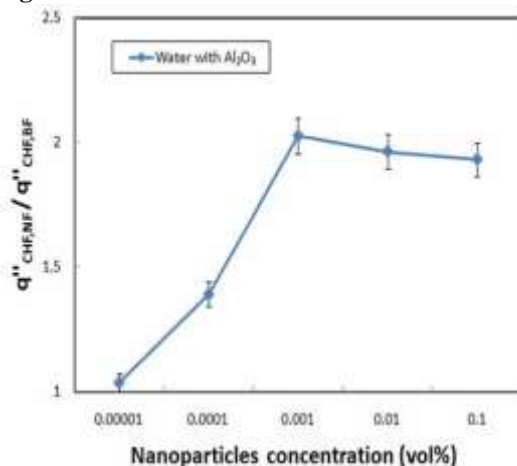
CHF characteristics in nano-fluids were investigated by R.N. Hegde et al. [1] with seven volume concentrations of CuO in water ranging from 0.01% to 0.5%. For nanofluid with a minimum concentration of 0.01%, the CHF was enhanced by 70% of the value of pure water while the CHF sharply increased up to 130% of the value of pure water at particle concentrations of 0.2 %, and then became saturated at about 60% at 0.3% volume concentration and onwards.

It was observed that surface roughness value (Ra value) increased from 0.09 μm to 0.23 μm corresponding to 0.05% volume and 0.3% volume concentrations of CuO nanofluid. Hence surface

modification was responsible for CHF enhancement [2].

H. Kim et al. [5] observed from pool boiling experiment that with the increased of the particle concentration, CHF of nanofluids in all cases (TiO<sub>2</sub> on NiCr wire, Al<sub>2</sub>O<sub>3</sub> on NiCr wire, and TiO<sub>2</sub> on Ti wire) sharply increased, and then becomes saturated with the significant CHF enhancements. CHF of water-TiO<sub>2</sub> nanofluids from a NiCr heater increases up to 148% of the value of pure water at particle concentrations below 10<sup>-3</sup> %, and then becomes saturated at about 200% at 10<sup>-1</sup>%. This tendency of CHF of water-TiO<sub>2</sub> nanofluids from a NiCr heater is very similar to CHF enhancement of water TiO<sub>2</sub> nanofluids from a Ti heater. On the other hand, CHF of water Al<sub>2</sub>O<sub>3</sub> nano-fluids on a NiCr heater has a little different behavior from the results of water-TiO<sub>2</sub> nano-fluids on NiCr and Ti wires. At the small concentration of 10<sup>-3</sup> %, the CHF enhancement is much higher than those of TiO<sub>2</sub> nanofluids and attains the maximum of 176% without more increase though increasing particle concentration to 10<sup>-1</sup> %. The reason for these results were microstructure and topography of the heater surface modified by the deposition of suspended nanoparticles during the pool boiling of nano-fluids. Characteristics of nanoparticle surface coating are highly dependent on the concentration of nano-fluids. In the case of nano-fluids with the lowest particle concentration, 0.00001% the heating surface displays only a nominal change. However, as the concentration increases, surface deposition of the nanoparticles thickens and more micro-sized structures are formed on the heating surface due to aggregation of nanoparticles. [5]

**Figure 3:**



*CHF enhancement ratio at various nanoparticle volume concentrations.*

Y. Jung et.al. [9] measured the critical heat flux of H<sub>2</sub>O/LiBr-based binary nanofluids having Al<sub>2</sub>O<sub>3</sub> of nanoparticle concentrations as 0.01, 0.03, 0.05 and 0.1 volume %. They found that CHF increases as nanoparticles concentration increases in base fluid. Highest CHF increased up to 48.5 % at 0.1 volume % concentration than base fluid. It is attributed that the CHF is increased by the surface modifications of the heater due to the deposition of nanoparticles during the boiling on it and the reduction of contact angles leads to increase in wettability.

With increase in concentration of SiO<sub>2</sub> in DI water from 0.1 volume% to 2 volume %, D.M. Vazquez, R. Kumar [10] obtained CHF enhancement compared to pure water having maximum value between 0.2 volume percent to 0.4 volume percent. Maximum CHF enhancement was 300%. As concentration is increased further, CHF enhancement decreases and attains 200% enhancement at 1 % volume concentration and very slight deviation is observed at 1.5 and 2 % volume concentration. Deposition was observed for all concentrations with a general upward trend. High heat transfer through interaggregate pores and the highly wettable nature of the nanosilica are the main factors in CHF enhancement, with declining effect as deposition increases, lending itself to a detrimental thermal resistance effect.

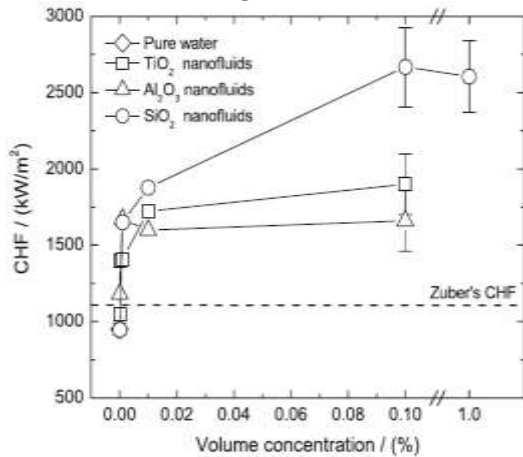
R. Kathiravan et al. [11] in their experiment varied Cu nanoparticles in distilled water as 0.25%, 0.5% and 1 % weight concentration. Result showed that at concentration 0.25%, 0.5% and 1% CHF increased by 25%, 40% 48% respectively.

E.S. Kim et al. [12] used Al<sub>2</sub>O<sub>3</sub> nanoparticle concentrations in water in range from 0.00001 to 0.1 % volume concentrations. The CHF increases up to 103% (compared to pure water) as Al<sub>2</sub>O<sub>3</sub> particle concentration increases until 0.001 volume % while it starts to decrease gradually as the particle concentration increases more than 0.001 volume % as shown in figure 3.

Figure 4 shows [4] the pool boiling CHF values of various aqueous nanofluids with various particle volume concentrations. H. Kim and M. Kim found at concentrations below 0.01%, the CHFs of all of the nanofluids increased steeply up to about 170% of the value for pure water with increasing nanoparticle concentration. Increasing the concentration to 0.1%, no sizable increase in the CHF was observed for TiO<sub>2</sub>- and Al<sub>2</sub>O<sub>3</sub>- water nanofluids, while a considerable increase was observed for SiO<sub>2</sub>-water nanofluids. But water-SiO<sub>2</sub> nanofluids did not make

additional improvement in CHF against further increase of concentration to 1.0%. Therefore all nanofluids including water-SiO<sub>2</sub> nanofluids seem to have the same characteristics of CHF enhancement qualitatively.

Figure 4:



CHF at various nanoparticle volume concentrations.

The effect is strongly dependent on the nanoparticle material, as well as its concentration [37]. CHF enhancement occurs upto certain concentration of nanoparticle beyond which increase in concentration, saturation of CHF or decrement in CHF occurs depending upon surface morphology.

**Stabilizer effect on CHF**

J.Y. Jung et al. [9] in their experiment added only PVA to a 10 % weight concentration to LiBr in order to confirm the effect of PVA on CHF. It was found that PVA gives almost no effect on CHF in the pool boiling experiment.

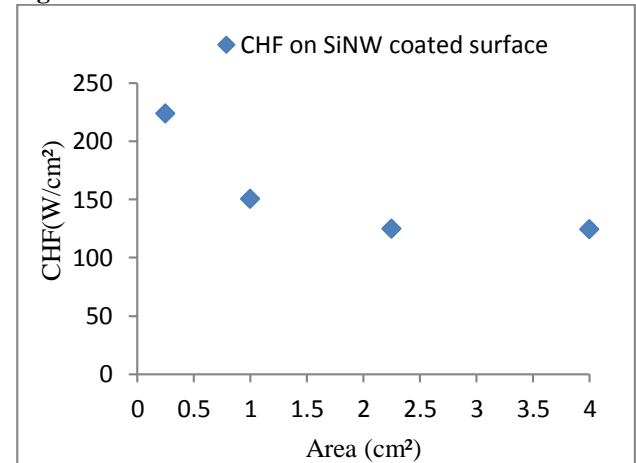
R. Kathiravan et al. [11] added 9.0 weight% of sodium lauryl sulphate anionic surfactant (SDS) in distilled water and Cu-distilled water nanofluids. CHF of distilled water with surfactant reduced by 81.51 % compared to CHF of distilled water. Addition of 9 % surfactant to Cu-distilled water nanofluids with weight concentration of 0.25%, 0.5% and 1 % CHF was reduced by 75 %, 68% and 62 respectively. Very less studies on stabilizer effect on nanofluids are there. From these studies exact effect cannot be explained. Hence more experimental work is expected in Stabilizer effect on CHF needs.

**Effect of heater size and geometry on CHF**

The effects of heater size on pool boiling CHF have been studied by M.C. Lu et al. [13] by using heater of sizes 0.5×0.5 cm<sup>2</sup>, 1×1 cm<sup>2</sup>, 1.5×1.5 cm<sup>2</sup> and 2×2 cm<sup>2</sup>. They obtained CHF on plain Si surfaces and

surfaces covered with a dense array of Si nanowires (SiNW). Highest CHF observed on SiNW covered and plain Si surfaces on a heater of size of 0.5×0.5 cm<sup>2</sup> among all heater sizes. The trend obtained of CHF on plain Si surfaces and SiNW array-coated surfaces with four different sizes of heaters are shown in Fig.5. The CHF on SiNW covered and plain Si surfaces for different sizes of heaters clearly indicate that the CHF increases as heater size reduces.

Figure 5:

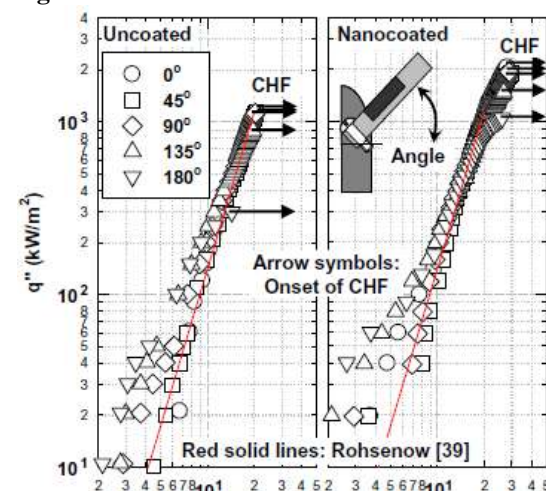


CHF on SiNW coated surface for different heaters sizes.

D.M. Vazquez and R. Kumar [10] noticed decreasing trend of the average CHF with increasing convective surface area of ribbon heaters. Considering the CHF on smallest and largest ribbons used in experimentations, for an 80% reduction in surface area, the data shows a 28% increase in average CHF.

**Effect of heater surface orientation**

Figure 6:



Effect of inclination angle of heater surface on CHF of pure water.

The effect of heater surface orientation on CHF of pure water was observed by S.M. Kwark et al. [14] conducting experiments over uncoated and nanocoated surface. Figure 6 shows Effect of inclination angle of heater surface on CHF of pure water. For both uncoated and nanocoated surfaces, as the inclination angle increases from 0° to 180°, the CHF values decrease. From 0° to 90°, the effect of inclination angle on CHF seems marginal. But as the inclination angle increases beyond 90° (135° and 180°), the CHF values decrease dramatically. Because beyond 90°, the bubbles cannot detach freely due to the blockage of the heated surface. The bubble residence time against the heated surface therefore increases. As a result, the bubbles flatten, merge with each other forming vapor blanket on the heater surface. This longer dwelling of a vapor blanket results in reaching CHF sooner for inclination angles beyond 90°.

I.C. Bang and S.H. Chang [15] investigated CHF for the both horizontal and vertical test section, in pool boiling of Alumina water nanofluid compared to water. For the horizontal test section 32% of CHF increased while for a vertical test section 13% of CHF increased. In case of horizontal heater bubbles can detach freely from surface than inclined heater surface. Hence, In pool boiling on horizontal heater shows more CHF enhancement than inclined heater.

## CONCLUSION

Over past decades many researcher work on nucleate pool boiling CHF enhancement. It is known that CHF of base fluid can be enhanced by suspending nanoparticles at small concentration. In this paper available studies reviewed and key causes of CHF enhancement are discussed.

- CHF of nanofluid depends on type of material of nanoparticles. In nanofluid pool boiling nanoparticles deposits on heater surface and form coating. Those nanoparticle material having shorter Rayleigh–Taylor wavelengths when coated on heater surface shows higher CHF enhancement while material having larger Rayleigh–Taylor wavelengths shows lower CHF enhancement.
- For nanoparticles size within nano range there is no significant variation in CHF. But comparing particle size within nano and micro range there is significant enhancement in CHF for particle size in nano range than particle size in micro range.

- With increasing nanoparticle concentrations CHF enhancement occurs up to certain concentration then CHF enhancement decreases with increase in nanoparticle concentration. Concentration affect heater surface. Researchers mentioned nanoparticle depositions on heater surface are responsible for CHF enhancement. Detail effect of nanoparticles concentrations on surface modifications needs to be find out which leads to CHF enhancement.
- Stabilizer like PVA shows no effect on CHF enhancement but CHF of nanofluid with anodic surfactant (SDS) severely diminished. Hence more work is required to understand the effect of stabilizer on CHF.
- CHF strongly depends on heater size and geometry. CHF found higher on smaller heater size, also CHF on cylindrical wires is clearly higher than that of the ribbons heaters.
- CHF values are higher for horizontal heater than inclined or vertical. CHF decreases as angle of inclination of heater increases.

## FUTURE WORK

From review of previous studies it is found that nanoparticle depositions on heater surface are responsible for CHF enhancement. But these depositions on heater surface will be different at different nanoparticle concentrations. Hence surface roughness of heater surface will be different at different concentrations. Detail study is required on Effect of nanoparticle concentrations on Surface roughness and its effect on CHF. Also researchers have taken large nanoparticle concentration range for investigations. Even small amount variations in concentration can significantly affect CHF of nanofluid. Thus in order to utilized nanofluid in practical applications complete database is required considering influence of above discussed parameters as well as heater surface modifications. Hence vast investigation is required in the area of CHF enhancement using nanofluid.

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